64GFC 57.8Gb/s PAM-4 FEC Capabilities

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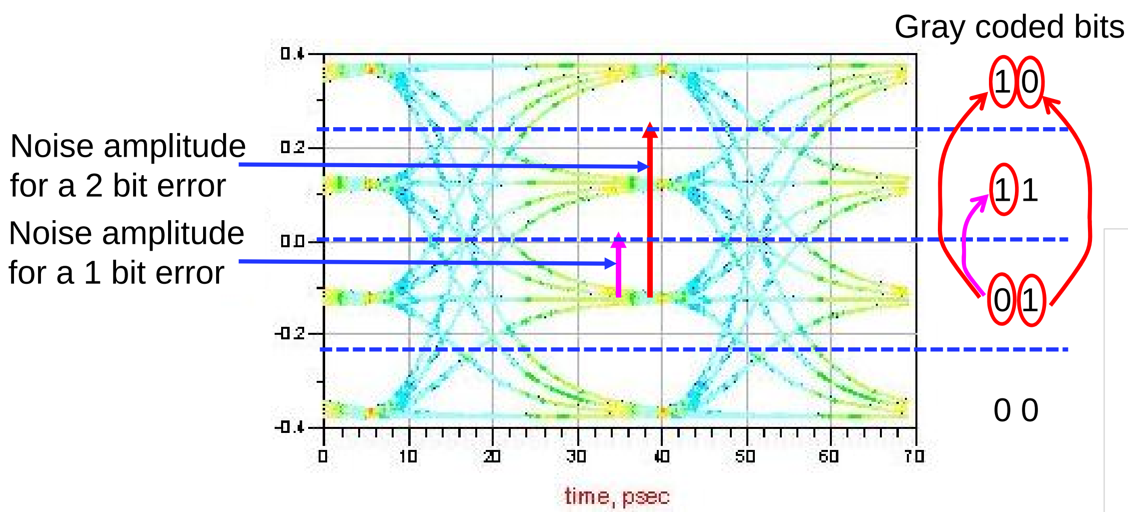
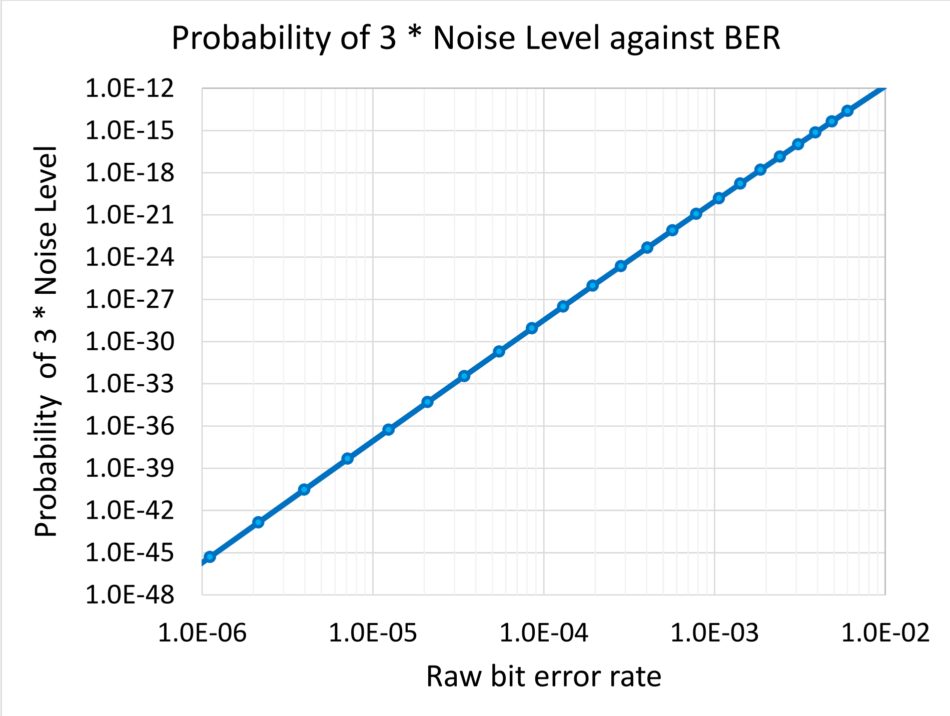
Purpose of this work

- To determine if the (544,514,10) RS FEC allows any relaxation of the BER requirement of 1e-6 for the 64GFC standard optical link
- End to end link performance of 1e-15 BER is required after FEC correction

- It follows the flow in the IEEE presentation of Anslow_3bs_03_5015

Gray coded PAM-4 bit error rate probabilities





For a single bit in error the noise amplitude is ~0.5 * level spacing

For a 2 bit error the noise amplitude is ~1.5 * level spacing, so = 3 * noise amplitude for a single bit error assuming no linearity error

Probability of 2 bit error is < 1e-20 for 1e-3 BER and <1e-45 for 1e-6 BER, so 2 bit errors can be ignored for the rest of this presentation

Basic error probability theory

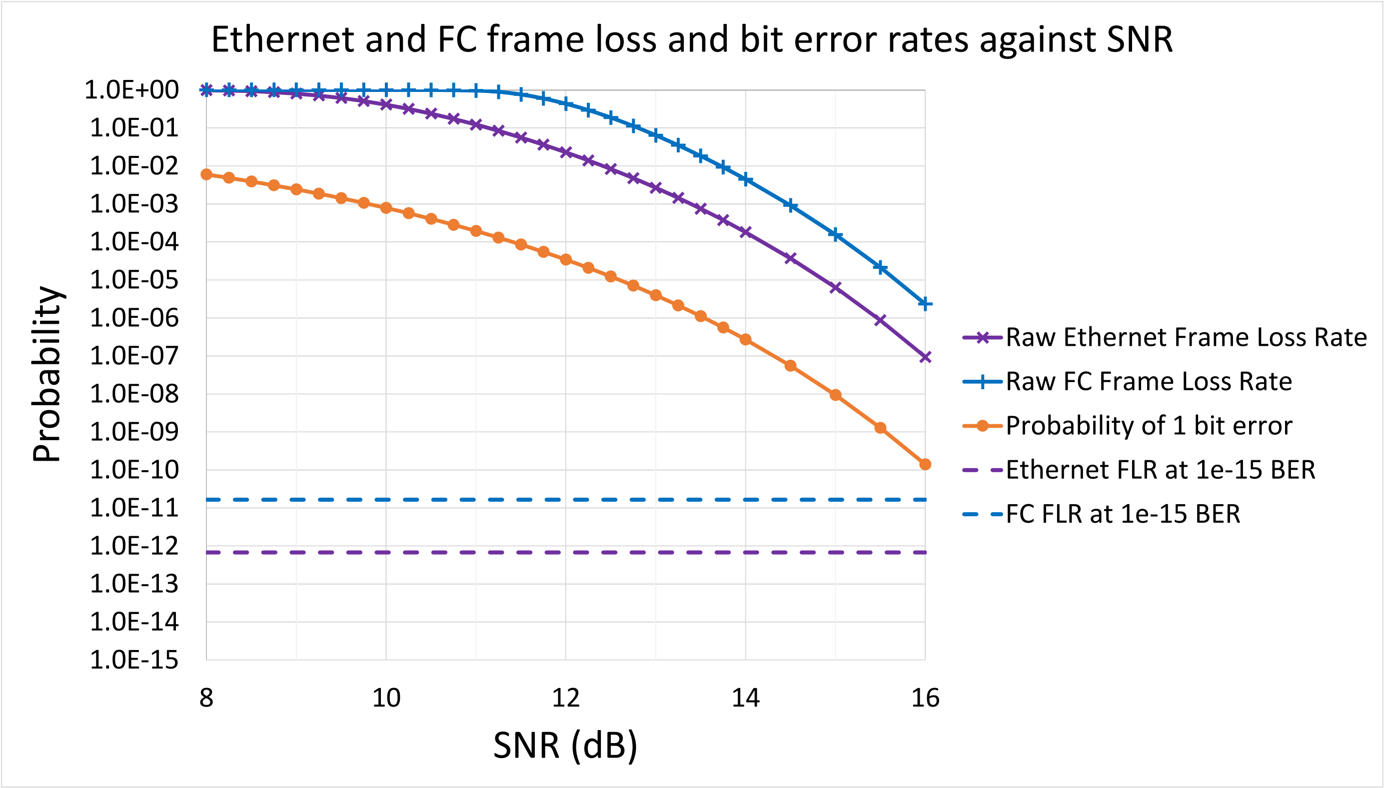
- The value of the Signal to Noise Ratio (SNR) used in the following curves is derived from the equation for a random noise induced error:

$$BER_{in} = \frac{1}{2} erfc\left(\frac{SNR}{\sqrt{2}}\right)$$

- Where SNR is the signal to noise ratio expressed as a ratio and not in dB
- The minimum Ethernet frame length is (64 + 8 + 12) * 8 = 672 bits and is used to correlate results with the Anslow presentation
- If the probability of a bit error is BER_{in} then the probability of a bit not being in error is $(1 - BER_{in})$
- Therefore, if we assume no error propagation, then the probability of a frame of 672 bits having at least one error is $1 - (1 - BER_{in})^{672}$ and is the Ethernet Frame Loss Ratio (FLR)

NRZ Random errors with no error propagation with no FEC

These curves use the minimum Ethernet frame size of 84 bytes (64 + 8 + 12) and the typical FC frame size of 2082 bytes (1 + 24 + 2048 + 4 + 4 + 1)



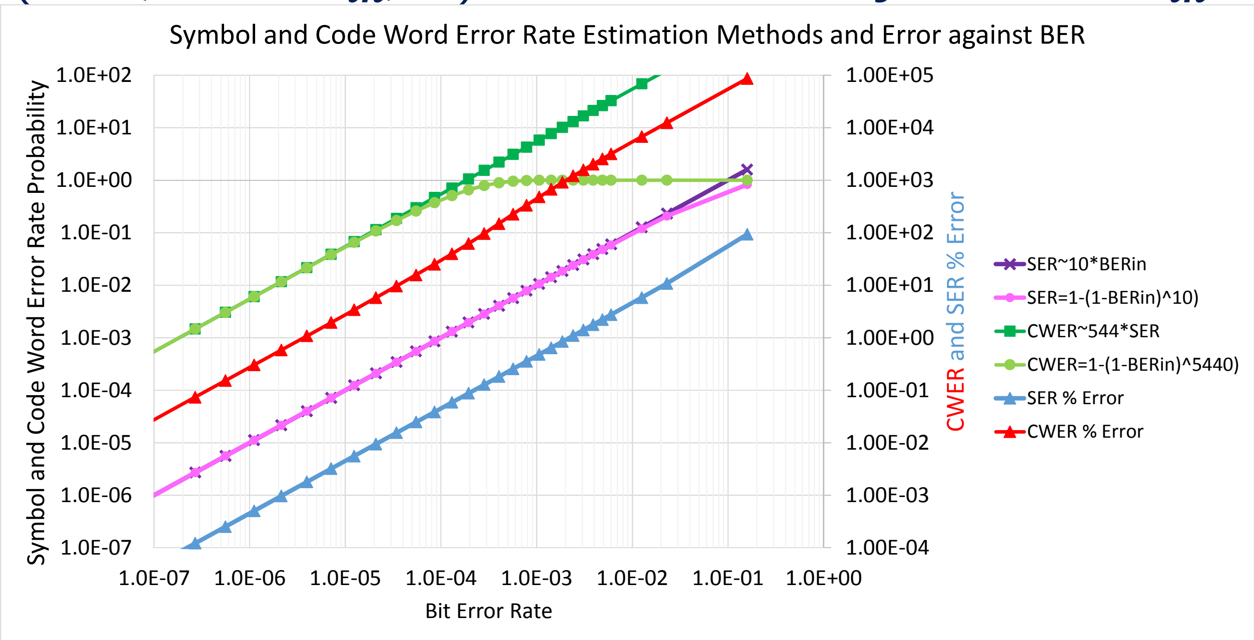
(544,514,10) RS FEC error correction capability

- Block length n = 544
- Message length k = 514
- Symbol length = 10
- Hamming distance = $n - k$ +1 = 31
- Erroneous symbols in a block that can be corrected = 0.5 * ($n - k$) = 15

NRZ FEC mathematics with no error propagation

- **For a BER of BER_{in} the probability of 1 bit not having an error is $(1 - BER_{in})$**
- **The probability of a 10 bit symbol not having an error is $(1 - BER_{in})^{10}$**
- **The probability of a 10 bit symbol having any number of errors is:**

$\left(1 - (1 - BER_{in})^{10}\right)$ = SER, for a very small BER_{in} this reduces to $10 * BER_{in}$



How small can we use the approximation?
As we have 544 symbols per code word the probability of no errors in a code word is
$(1\text{-Symbol Error Rate})^{544}$
The simple approximation would be 5440*BER and only gives less than 1% error for BER < ~3e-6 and >100% error for 3e-4 and above, so not good for the BER range we need to consider

SER = Symbol error rate
CWER = Code word error rate

FEC mathematics with no error propagation – continued

- The probability of an error free symbol $P(0) = 1 - SER$
- The probability of 544 error free symbols $P(544, no\ errors) = (1 - SER)^{544}$

- The probability of one errored symbol $P(1) = 1 - (1 - BER_{in})^{10} = SER$ $\{= 1 - (1 - DER_{in})^{5}$ for PAM-4$\}$
- Number of ways of picking one errored symbol from 544 symbols $= 544$
- The probability of one errored symbol in 544 symbols $P(1, 544) = 544 * SER$
- The probability of 543 symbols with no errors $P(543, no\ errors) = (1 - SER)^{543}$
- The probability of one errored symbol and 543 non-errored symbols

$$P(1\ error\ only) = 544 * SER * (1 - SER)^{543}$$

- The probability of two errored symbols in 2 symbols $P(2) = SER^2$
- Number of ways of picking 2 symbols from 544 $= \frac{544!}{(544-2)! * 2!}$
- Probability of 2 errored symbols in 544 $= \frac{544! * SER^2}{(544-2)! * 2!}$
- The probability of *542* symbols with no errors $P(542, no\ errors) = (1 - SER)^{542}$
- The probability of two errored symbols and 542 non-errored symbols

$$P(2\ errors\ only) = \frac{544! * SER^2}{(544-2)! * 2!} * (1 - SER)^{542}$$

FEC mathematics with no error propagation – continued 2

- The probability of n errored symbols $P(n) = SER^n$
- Number of ways of picking n errored symbols from 544 $= \frac{544!}{(544-n)!n!}$
- The probability of *544-n* symbols with no errors

  $$P(544 - n, no\ errors) = (1 - SER)^{544-n}$$

- Probability of n errored symbols in 544 $= \frac{544! * SER^n}{(544-n)! * n!}$
- The probability of n errored symbols and *544-n* non-errored symbols

  $$P(n\ errors\ only) = \frac{544! * SER^n}{(544-n)! * n!} * (1 - SER)^{544-n}$$

FEC mathematics with no error propagation –continued 3

- The probability of up to 15 errored symbols

$$P(0to15\ errors) = \sum_{n=0}^{n=15} P(n\ error\ only) = \sum_{n=0}^{n=15} \frac{544! * SER^n}{(544-n)! * n!} * (1-SER)^{544-n}$$

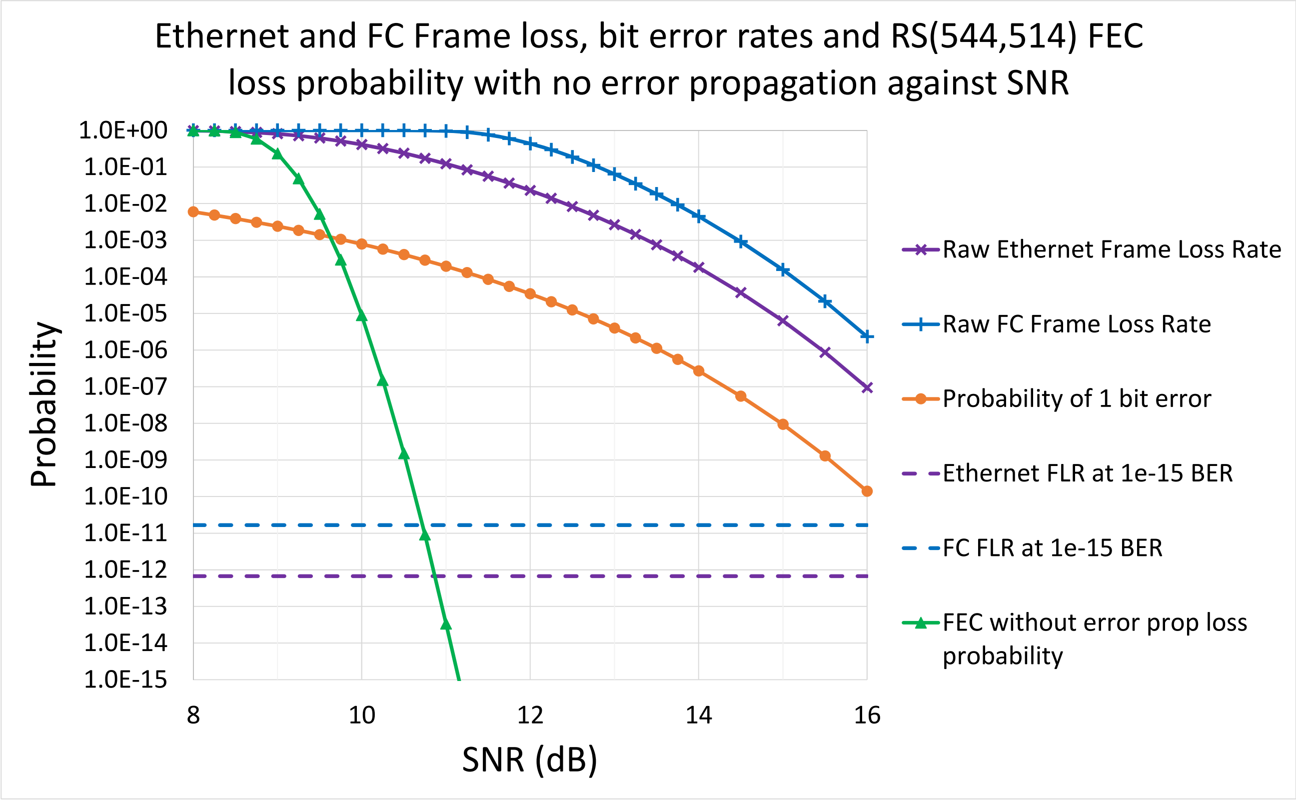
- All of the 0 to 15 errored symbols cases will be corrected by the FEC, so the probability of an uncorrected error will be

$$P(FECuncorrected) = (1 - P(0to15\ errors))$$

$$P(FECuncorrectable) = 1 - \sum_{n=0}^{n=15} \frac{544! * SER^n}{(544-n)! * n!} * (1-SER)^{544-n}$$

NRZ Random errors with no error propagation with and without FEC

These curves use the minimum Ethernet frame size of 84 bytes (64 + 8 + 12) and the typical FC frame size of 2082 bytes (1 + 24 + 2048 + 4 + 4 + 1)

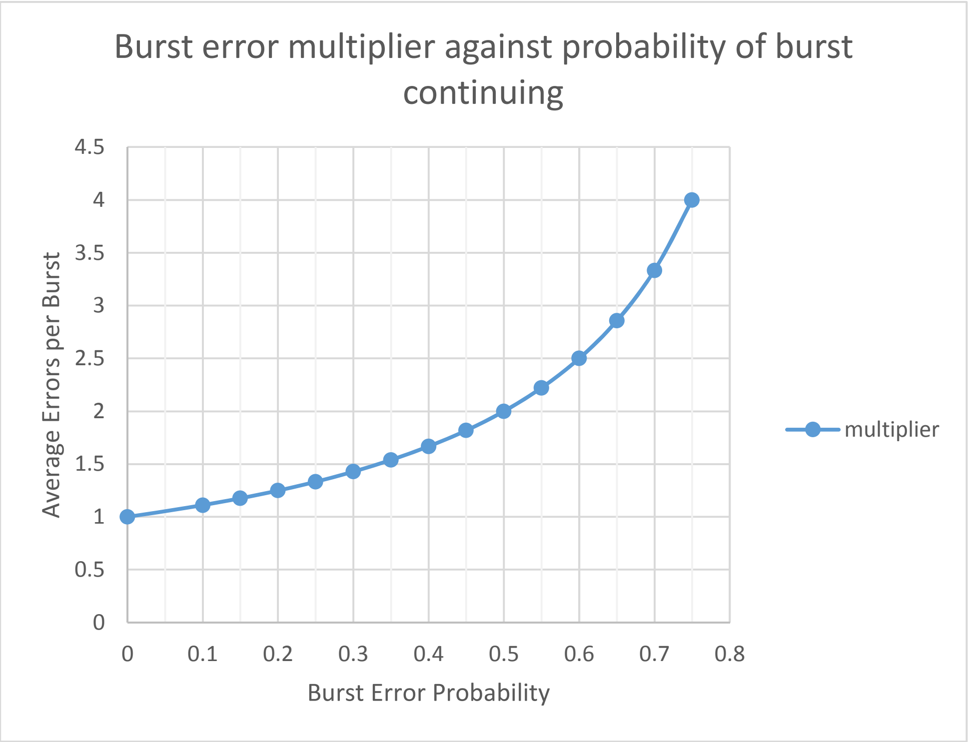


Effect of Burst Error Probability on BER

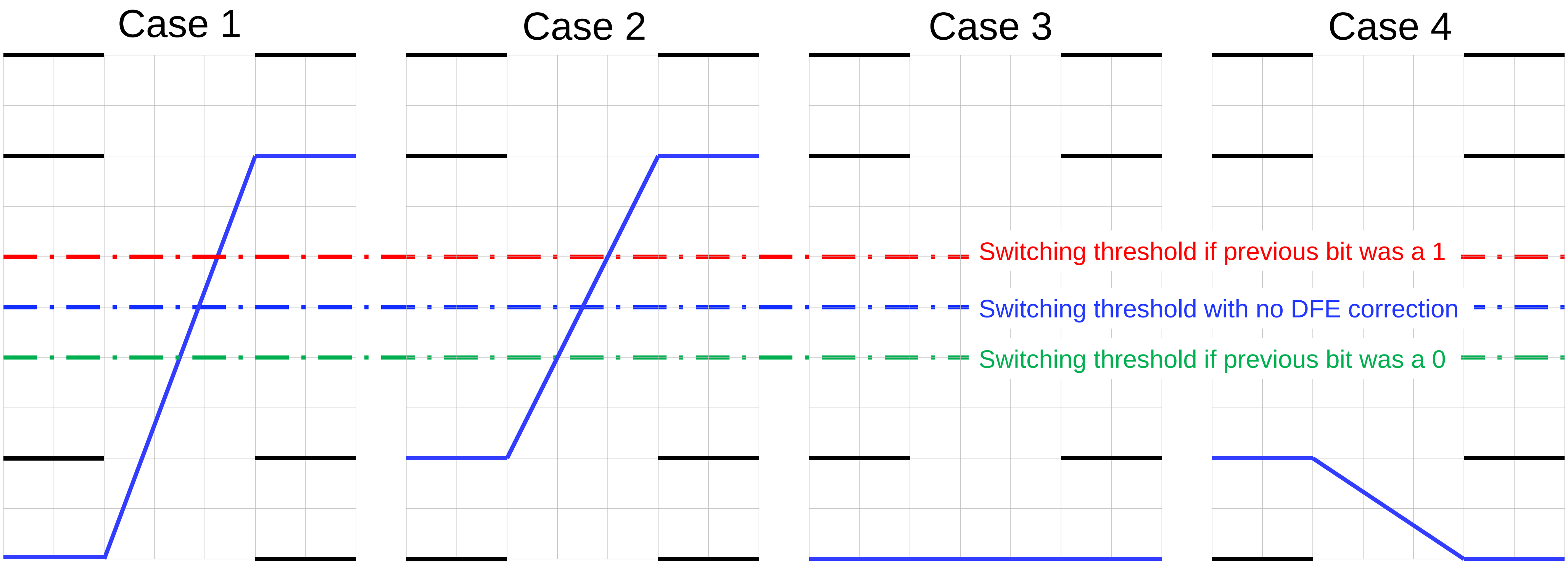
- **The average number of errors in a burst is simply related to the probability of an error propagating (A)**

$$Average\ errors\ per\ burst = \frac{1}{1-A}$$

- **However, with PAM-4 there are only 5 PAM-4 symbols per 10 bit FEC symbol**
- **For an error in the last bit of one FEC symbol with PAM-4 to propagate into the next symbol is very high, but to propagate to the following FEC symbol after that is also high and needs to be considered**



NRZ Error propagation options for 1 DFE tap only



0 to 1 (prior bit 0)
If correct margin = 4
If incorrect margin = 2

0 to 1 (prior bit 1)
If correct margin = 4
If incorrect margin = 2

0 to 0 (prior bit 0)
If correct margin = 4
If incorrect margin = 6

0 to 0 (prior bit 1)
If correct margin = 4
If incorrect margin = 6

In 50% of all cases the margin is reduced by 2 * DFE tap 1 value and could cause an error to propagate

Worst Case PAM-4 Error Propagation

Correct level	Perceived RX level		Errored bits	
	One higher	One lower	One higher	One lower
3 (1,0)	3 (1,0)	2 (1,1)	✓, ✓	✓,×
2 (1,1)	3 (1,0)	1 (0,1)	✓,×	×, ✓
1 (0,1)	2 (1,1)	0 (0,0)	×, ✓	✓,×
0 (0,0)	1 (0,1)	0 (0,0)	✓,×	✓, ✓

- **Two of these cases result in ✓, ✓ so the error will not propagate (25%)**
- **This means a 75% chance of a PAM-4 error propagating if the DFE taps add up to 1 level change in the PAM-4 symbol**
- **For this case the average burst length = 4 PAM-4 symbols**
- **A more realistic worst case would be to assume a 50% chance of the PAM-4 error propagating and this gives an average burst length = 2 PAM-4 symbols**
- **Two of these cases result in errors in the first bit (25%)**
- **Four cases in the second bit (50%)**

FEC Symbol Error Propagation for PAM-4 Errors

The worst case probability of an error in the first PAM-4 symbol affecting the second PAM-4 symbol is 0.75, the third PAM-4 symbol is 0.75^2, the fourth is 0.75^3, the fifth 0.75^4 and the first PAM-4 symbol in the next FEC symbol is 0.75^5.

This can be done for the second, third, fourth and fifth PAM-4 symbols being in error and gives the total probability of an error in one FEC symbol causing the next FEC symbol to also be in error of:

$$P_{err}(next\ FEC\ symbol) = \frac{(0.75^5 + 0.75^4 + 0.75^3 + 0.75^2 + 0.75^1)}{5} = 0.4576$$

This gives a 46% chance of the next FEC symbol being in error due to a burst of errors

The alarming extension of this is that the next but one FEC symbol being in error has a probability given by:

$$P_{err}(next\ but\ one\ FEC\ symbol) = \frac{(0.75^{10} + 0.75^9 + 0.75^8 + 0.75^7 + 0.75^6)}{5} = 0.1086$$

$$P_{err}(next\ but\ one\ FEC\ symbol) = 0.75^5 * P_{err}(next\ FEC\ symbol)$$ Note $0.75^5 = 0.2373$

FEC Symbol Error Propagation for PAM-4 Errors - continued

- **This can be further extended to give:**

$$P_{err}(next\ but\ \boldsymbol{n+1}\ FEC\ symbol) = 0.75^5 * P_{err}(next\ but\ \boldsymbol{n}\ FEC\ symbol)$$

The probability of a given FEC symbol being errored due to error propagation

$$SEP_{err}(due\ to\ error\ prop) = SER * 0.4576 * (1 + 0.75^5 + 0.75^{5*2} + 0.75^{5*3} \ldots)$$

$$SEP_{err}(due\ to\ prop) = \frac{0.4576*SER}{(1-0.75^5)} \cong SER * 0.6000 \text{ for A=0.75}$$

Number of symbols error propagates	Probability	Number of symbols error propagates	Probability	Number of symbols error propagates	Probability	Number of symbols error propagates	Probability
1	0.4576	5	1.45E-03	9	4.60E-06	13	1.46E-08
2	0.1086	6	3.44E-04	10	1.09E-06	14	3.46E-09
3	0.02577	7	8.17E-05	11	2.59E-07	15	8.22E-10
4	6.12E-03	8	1.94E-05	12	6.15E-08	16	1.95E-10

FEC Symbol Error Propagation for PAM-4 Errors for A=0.5

A more realistic worst case probability of an error in the first PAM-4 symbol affecting the second PAM-4 symbol is 0.5, the third PAM-4 symbol is 0.5^2, the fourth is 0.5^3, the fifth 0.5^4 and the first PAM-4 symbol in the next FEC symbol is 0.5^5.

This gives the total probability of an error in one FEC symbol causing the next FEC symbol to also be in error of:

$$P_{err}(next\ FEC\ symbol) = \frac{(0.5^5 + 0.5^4 + 0.5^3 + 0.5^2 + 0.5^1)}{5} = 0.19375$$

This is ~20% chance of the next FEC symbol being in error due to a burst of errors

The extension of this is that the next but one FEC symbol being in error has a probability given by:

$$P_{err}(next\ but\ one\ FEC\ symbol) = \frac{(0.5^{10} + 0.5^9 + 0.5^8 + 0.5^7 + 0.5^6)}{5} = 6.05\text{e} - 3$$

$$P_{err}(next\ but\ one\ FEC\ symbol) = 0.5^5 * P_{err}(next\ FEC\ symbol)$$ Note $0.5^5 = 1/32$

FEC Symbol Error Propagation for PAM-4 Errors - continued

- **This can be further extended to give:**

$$P_{err}(next\ but\ \mathbf{n}+\mathbf{1}\ FEC\ symbol) = 0.5^5 * P_{err}(next\ but\ \mathbf{n}\ FEC\ symbol)$$

The probability of a given FEC symbol being errored due to error propagation

$$SEP_{err}(due\ to\ error\ prop) = SER * 0.19375 * (1 + 0.5^5 + 0.5^{5*2} + 0.5^{5*3} \ldots)$$

$$SEP_{err}(due\ to\ prop) = \frac{0.19375*SER}{(1-0.5^5)} \cong SER * 0.2 \text{ for A=0.5}$$

Number of symbols error propagates	Probability	Number of symbols error propagates	Probability	Number of symbols error propagates	Probability	Number of symbols error propagates	Probability
1	0.19375	5	1.85E-07	9	1.76E-13	13	1.68E-19
2	6.05e-3	6	5.77E-09	10	5.51E-15	14	5.25E-21
3	1.89e-4	7	1.80E-10	11	1.72E-16	15	1.64E-22
4	5.91E-06	8	5.64E-12	12	5.38E-18	16	5.13E-24

Calculation of the Probability of Random Errors Propagating to Break the FEC

For 2 random errors combine the probabilities of the error propagation combinations: 1 of the random errors propagates ≥14 bits breaks the FEC, or 1 error propagates ≥13 and the other error propagates ≥1, or 1 error propagates ≥12 and the other ≥2, and so on etc.

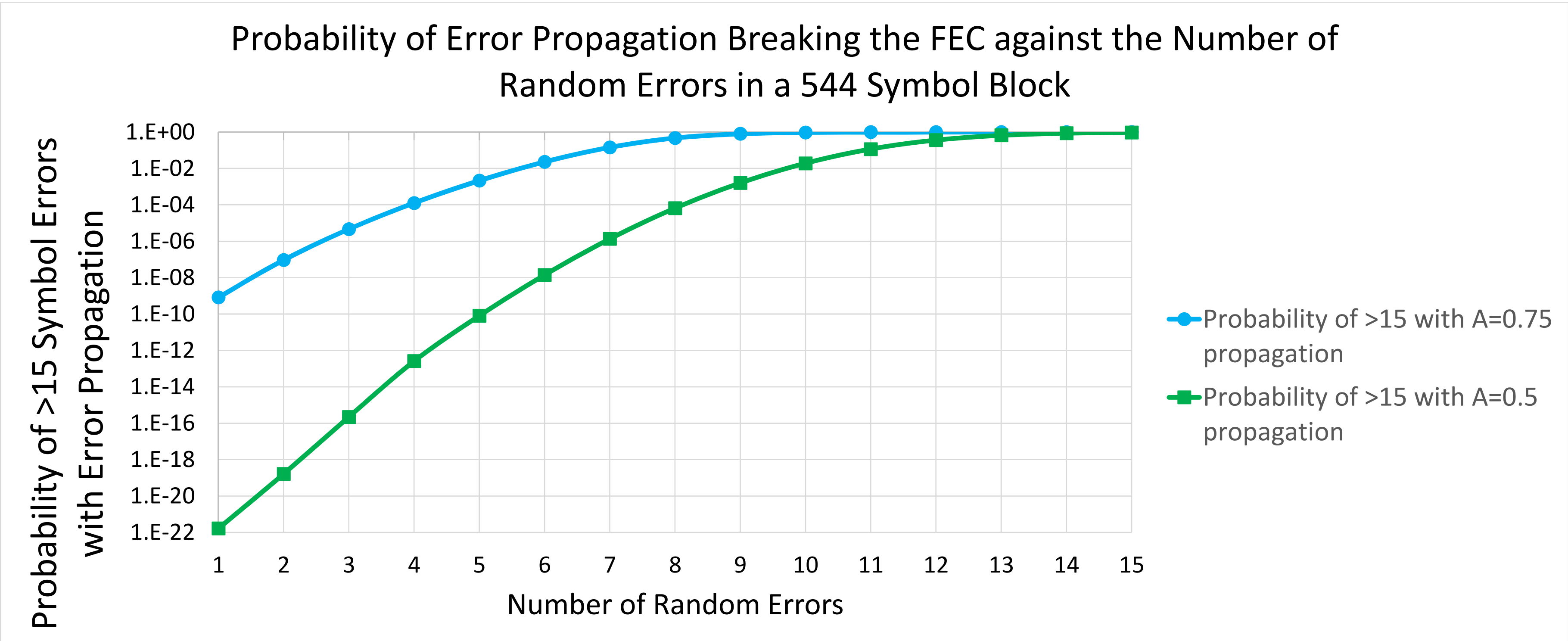
$$2 * \{P(14) + P(13) * P(1) + P(12) * P(2) + P(11) * P(3) + P(10) * P(4) + P(9) * P(5) + P(8) * P(6)\} + P(7) * P(7)\} = PEPbreaksFEC(2) = 9.37\mathrm{e} - 8 \; for\; A = 0.75 \text{ and } 1.64\mathrm{e} - 19 \text{ for } \mathrm{A} = 0.5$$

where *P*(*n*) is the probability a random symbol error will propagate *n* symbols and the factor of two is because which of the two errors is propagating *m* and *n* symbols does not matter

This calculation is then repeated for 3 random errors and so on all the way up to 15 random errors and the probabilities are added up as:

$$\{1 - \{[1 - PEPbreaksFEC(2)] * [1 - PEPbreaksFEC(3)] \; \ldots * [1 - PEPbreaksFEC(15)\}\}$$

Probability of Random Errors Propagating to Break the FEC for A=0.75 and A=0.5



FEC Probabilities with Error Propagation

- **The probability of a random error free symbol** $P(0\ errors) = 1 - SER$
- **The probability of 544 random error free symbols** $P(544, no\ random\ errors) = (1 - SER)^{544}$
- **The probability of the previous symbol to the 544 symbols having a random error is SER**
- **The probability that an error propagates into the 544 random error free symbols from the previous 544 symbols is taken care of by not restricting the number of combinations to the current 544 symbols**
- **Probability of zero random and propagation errors not breaking the FEC in 544 symbols**
$$P(544, 0\ random\ or\ propagation\ errors) = (1 - SER)^{544}$$
- **The probability of one random errored symbol** $P(1) = 1 - (1 - DER_{in})^5$ **for PAM−4** $= SER$ **(DER=detector error rate and is the PAM-4 symbol error rate)**
- **The worst case probability of that error propagating to the next 15 symbols =** 8.22e-10 (see slide 16 for A=0.75, or slide 18 shows 1.68e-22 for A=0.5)
- **The probability that this error does not propagate ≥15 symbols to make the FEC fail =** 1- 8.22e-10 (for A=0.75)
- **The probability of one random errored symbol that does not cause the FEC to fail** $= (1 - 8.22e - 10) * SER$
- **Number of ways of picking one errored symbol from 544 symbols** $= 544$
- **The probability of one random errored symbol not causing FEC failure in 544 symbols**
$$P(1, 544) = 544 * (1 - 8.22e - 10) * SER$$
- **The probability of 543 symbols with no random errors** $P(543, no\ random\ errors) = (1 - SER)^{543}$
- **The probability of one random errored symbol that does not cause FEC to fail and 543 non-errored symbols**
$$P(1\ random\ \&\ propagation\ does\ not\ break\ FEC) = 544 * (1 - 8.22e - 10) * SER * (1 - SER)^{543}$$

FEC Probabilities with Error Propagation - continued

- **The probability of two random errored symbols in 2 symbols** $P(2) = SER^2$
- **The probability that either will propagate to break the FEC (16-2 symbols)**

$\{\mathbf{P}EPbreaksFEC(2)\} = 9.37e - 8$ (see plot on slide 20 for 2 random errors A=0.75)

- **The probability of 2 random errors and propagation does not break FEC** $= (1 - 9.37e - 8) * SER^2$
- **Number of ways of picking 2 symbols from 544** $= \frac{544!}{(544-2)! * 2!}$
- **Probability of 2 random errored symbols FEC OK in 544** $= \frac{544! * (1-9.37e-8) * SER^2}{(544-2)! * 2!}$
- **The probability of *542* symbols with no random errors** $P(542, no\ errors) = (1 - SER)^{542}$
- **The probability of two errored symbols and 542 non-random errored symbols, FEC good after propagation**

$$P(2\ random\ and\ FEC\ good) = \frac{544! * (1 - 9.37e - 8) * SER^2}{(544 - 2)! * 2!} * (1 - SER)^{542}$$

FEC mathematics with error propagation – continued 2

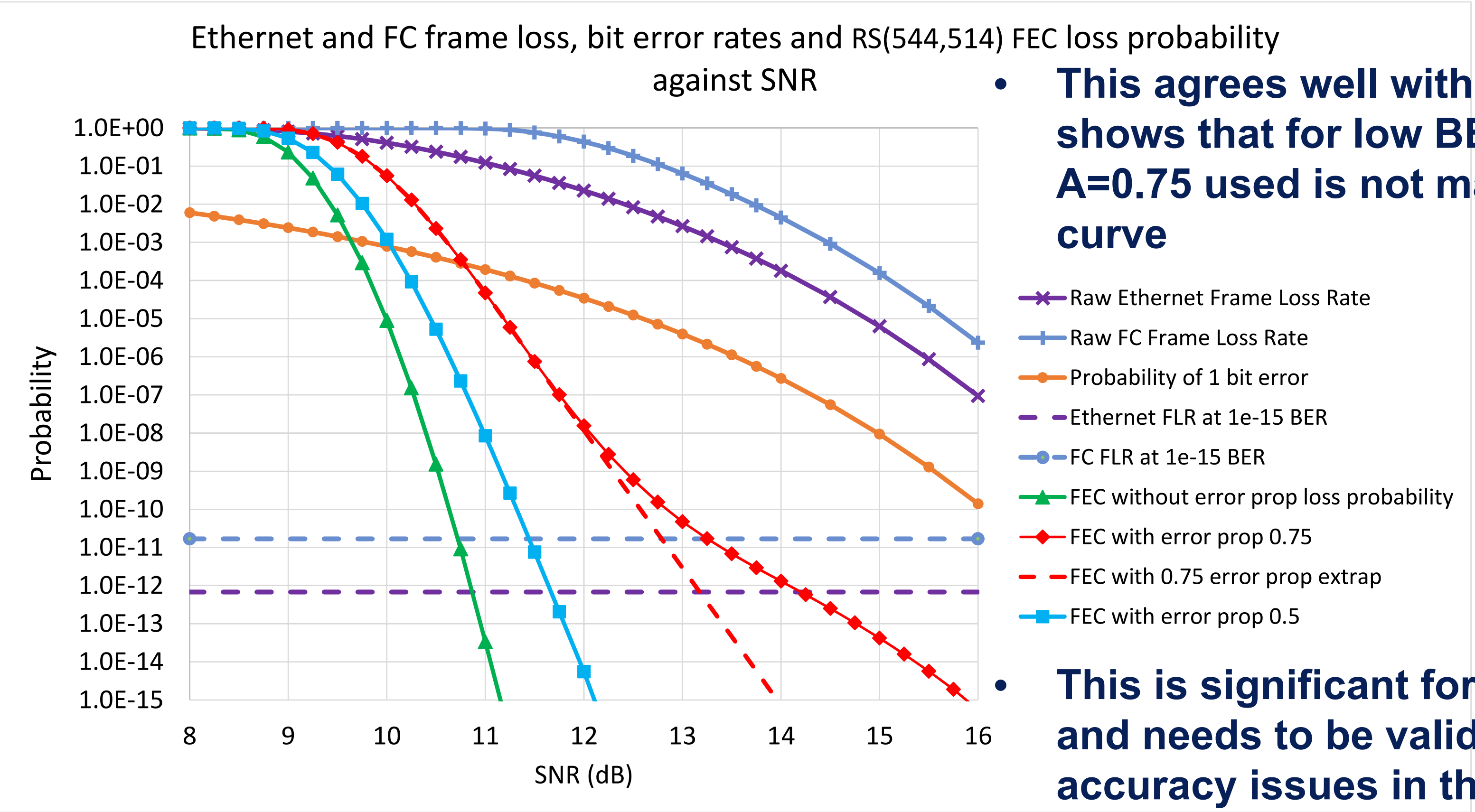
- The probability of random n errored symbols $P(n) = SER^n$
- Number of ways of picking n errored symbols from 544 $= \frac{544!}{(544-n)!n!}$
- The probability of $544-n$ symbols with no random errors

  $P(544-n, no\ errors) = (1-SER)^{544-n}$
- The probability of error propagation adding $16-n$ symbols with errors

  $PEPbreaksFEC(n) = \sum P(m+n+o+p \ldots \geq 16-n, errorprop)$ (slide 20)
- Probability of random n errored symbols in 544 $= \frac{544! * SER^n}{(544-n)! * n!}$
- The probability of n random errored symbols and $544-n$ non-errored random symbols FEC OK after error prop

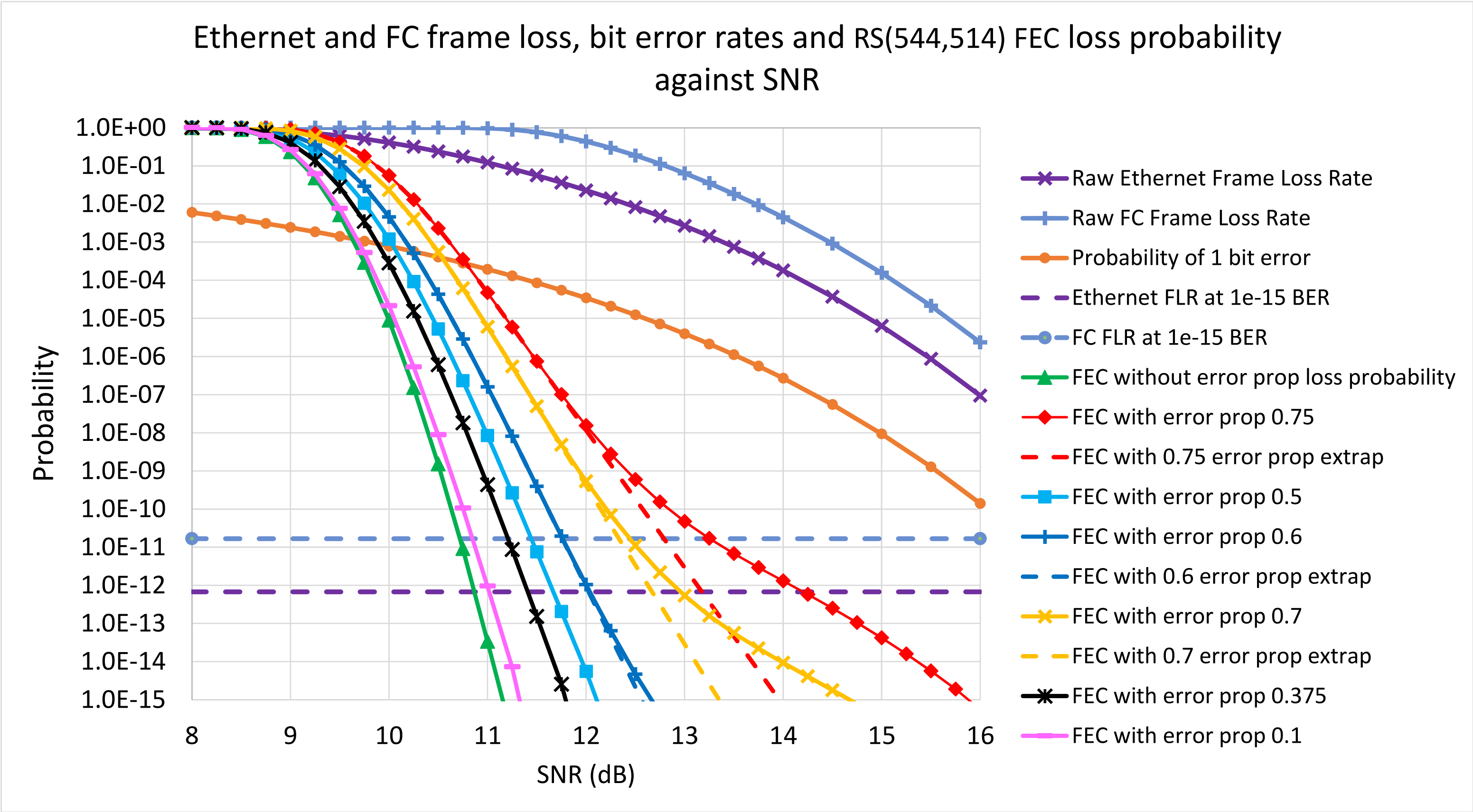
$$P(n\ errors, FEC\ OK) = (1 - PEPbreaksFEC(n)) * \frac{544! * SER^n}{(544-n)! * n!} * (1-SER)^{544-n}$$

Plot of Worst Case BER Capability with and without (544,514) FEC with PAM-4 with 0.5 and 0.75 Error Propagation Probability

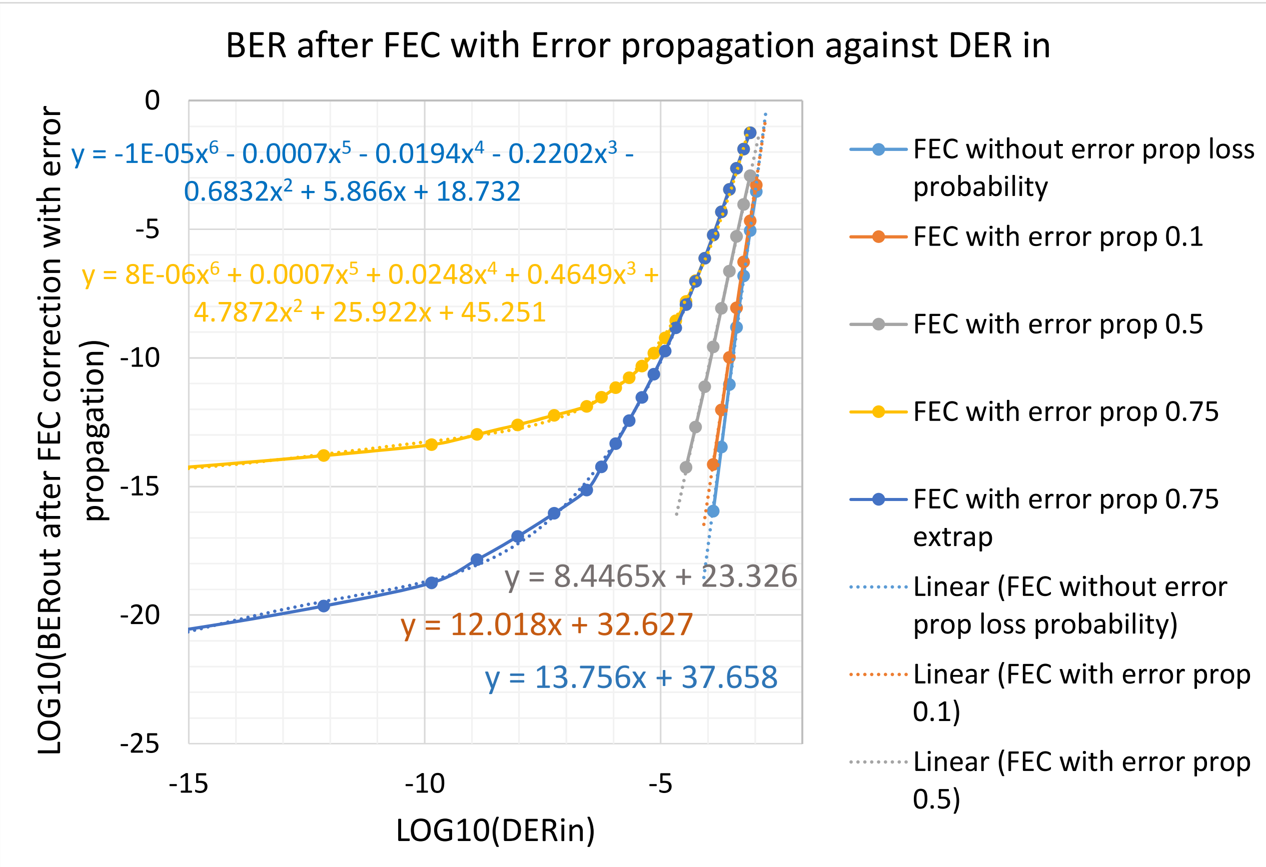


- **This agrees well with the Ethernet plots, but shows that for low BER the extrapolation for A=0.75 used is not matched by the theoretical curve**
- **This is significant for targets of 1e-15 BER and needs to be validated (it could be accuracy issues in the calculations)**

Plot of Worst Case BER Capability with and without (544,514) FEC with PAM-4 with Various Error Propagation Probabilities

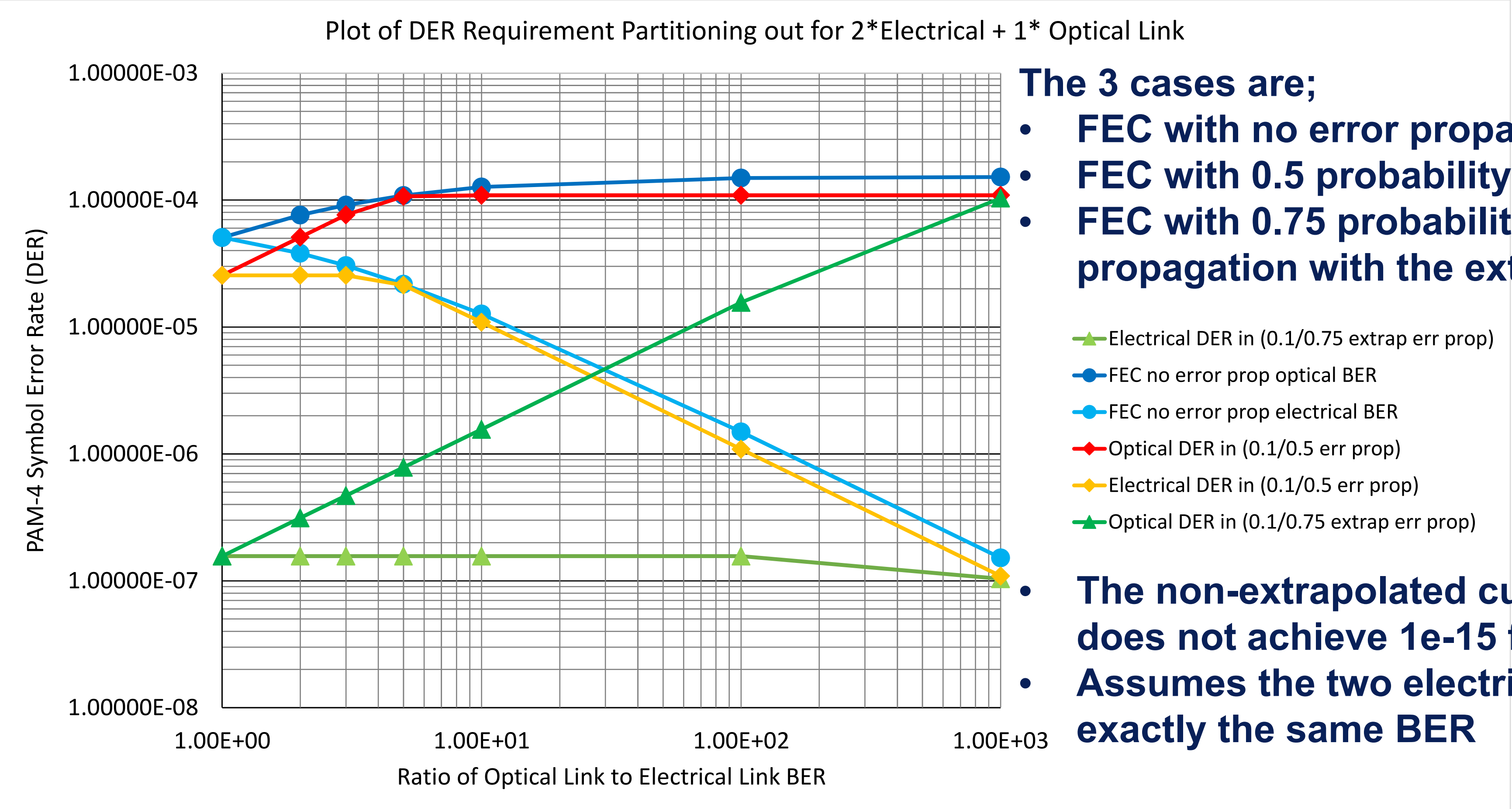


To Calculate the Link DER Budget Partitioning Optical/Electrical



- **Plot the LOG10(BERout after FEC) against LOG10(DERin)**
- **Perform a straight line or polynomial fit for ≤1e-3 DERin**
- **This allows the corrected FEC error rate to be calculated for any DERin**
- **Assume optical link has A=0 or 0.1**
- **Calculate sum of probabilities of each electrical and optical parts of the link and iterate until total BER <1e-15**
- **This does assume that error events on each of the 3 sections are independent (probability that an event on one part of the link overlaps an event on another part is not considered)**

Partitioning of Link BER between 2 electrical and 1 optical sections for an overall 1e-15 BER after error propagation & FEC



The 3 cases are;

- **FEC with no error propagation**
- **FEC with 0.5 probability error prop'n**
- **FEC with 0.75 probability error propagation with the extrapolated curve**

- **The non-extrapolated curve for A=0.75 does not achieve 1e-15 for 2 links**
- **Assumes the two electrical links have exactly the same BER**

Partitioning of Link DER budget – Interpretation of Results

Ratio of Optical / Electrical	No error propagation				A=0.1 optical, A=0.5 electrical error propagation				A=0.1 optical, A=0.75 extrapolated electrical error propagation				After FEC BER
	Optical DER in	Electrical DER in	Optical BER after FEC	Electrical BER after FEC	Optical DER in	Electrical DER in	Optical BER after FEC	Electrical BER after FEC	Optical DER in	Electrical DER in	Optical BER after FEC	Electrical BER after FEC	
1.00E-40	1.00E-40	1.51E-07	1.00E-40	7.61E-05	2.55E-20	2.55E-05	1.42E-203	5.00E-16	1.56E-22	1.56E-07	3.60E-230	5.00E-16	1.00E-15
1	1.01E-07	1.01E-07	5.07E-05	5.07E-05	2.55E-05	2.55E-05	2.66E-23	5.00E-16	1.56E-07	1.56E-07	6.74E-50	5.00E-16	1.00E-15
2	1.51E-07	7.57E-08	7.61E-05	3.80E-05	5.10E-05	2.55E-05	1.10E-19	5.00E-16	3.12E-07	1.56E-07	2.80E-46	5.00E-16	1.00E-15
3	1.82E-07	6.06E-08	9.13E-05	3.04E-05	7.64E-05	2.55E-05	1.41E-17	4.93E-16	4.69E-07	1.56E-07	3.66E-44	5.00E-16	1.00E-15
5	2.16E-07	4.33E-08	1.09E-04	2.17E-05	1.07E-04	2.13E-05	7.77E-16	1.11E-16	7.81E-07	1.56E-07	1.70E-41	5.00E-16	1.00E-15
10	2.52E-07	2.52E-08	1.27E-04	1.27E-05	1.09E-04	1.09E-05	9.99E-16	3.89E-19	1.56E-06	1.56E-07	7.03E-38	5.00E-16	1.00E-15
100	2.97E-07	2.97E-09	1.49E-04	1.49E-06	1.09E-04	1.09E-06	1.00E-15	1.52E-27	1.56E-05	1.56E-07	7.33E-26	5.00E-16	1.00E-15
1000	3.02E-07	3.02E-10	1.52E-04	1.52E-07	1.09E-04	1.09E-07	1.00E-15	5.90E-36	1.04E-04	1.04E-07	5.52E-16	2.24E-16	1.00E-15
1.00E+06	3.03E-07	3.03E-13	1.52E-04	1.52E-10	1.09E-04	1.09E-10	1.00E-15	3.49E-61	1.09E-04	1.09E-10	1.00E-15	2.03E-19	1.00E-15

Assuming the A=0.1 for the optical links and A=0.5 for the electrical links

- **If the electrical links and optical links are all considered to have 1/3rd the budget each (Ratio=1) then the DER requirement is ≤2.55e-5 for all links (random errors), or ≤2.83e-5 for the optical link and ≤5.10e-5 for each electrical link after error propagation**
- **If the optical and total electrical budget are assumed equal (Ratio=2) then the DER requirement is ≤5.10e-5 pre error prop (≤5.67e-5 post error prop) for the optical links and ≤2.55e-5 pre error prop (≤5.10e-5 post error prop) for each electrical link**
- **If the optical link has 10 times each electrical link DER (Ratio=10) then the DER requirement is ≤1.09e-4 pre error prop (≤1.21e-4 post error prop) for the optical links and ≤1.09e-5 pre error prop (≤2.18e-5 post error prop) for each electrical link**

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BER after FEC versus DERin for Various Error Propagation Probabilities