Preliminary FC64 57.8Gb/s PAM-4 FEC Capabilities
16-077v0

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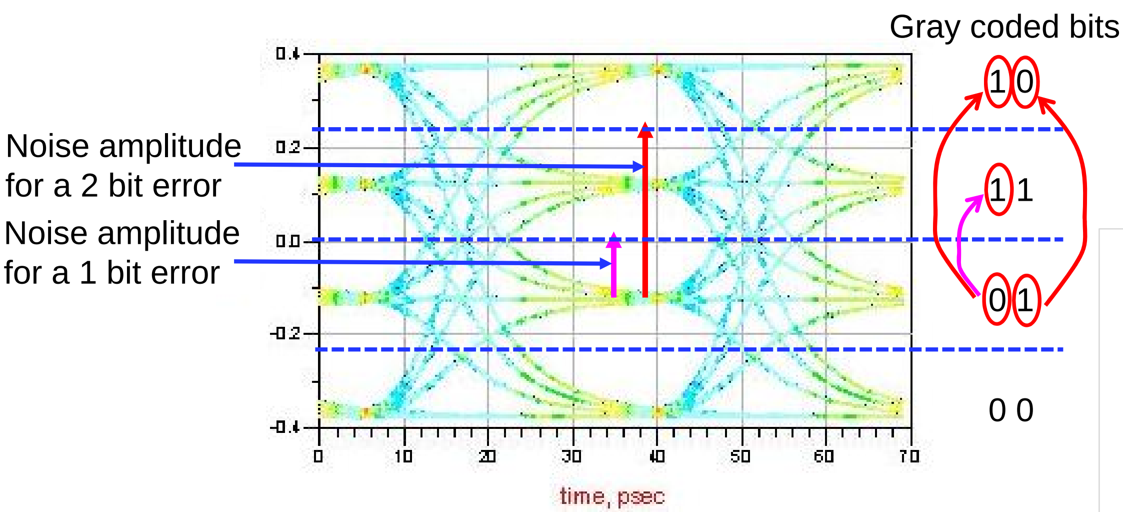
Purpose of this work

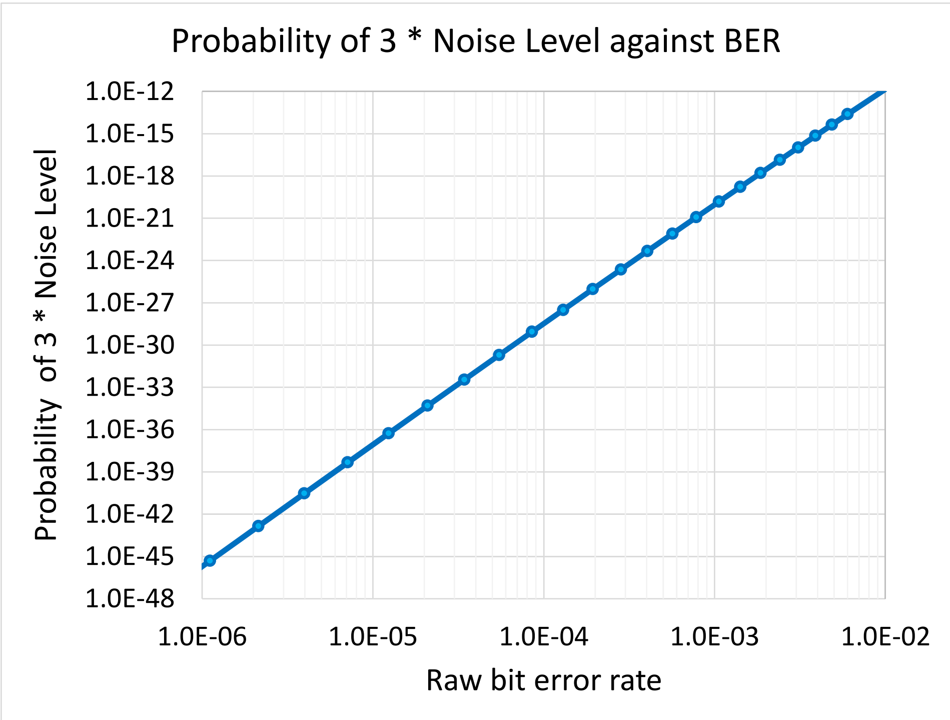
- To determine if the (544,514,10) RS FEC allows any relaxation of the BER requirement of 1e-6 for the FC-56 standard optical link
- End to end link performance of 1e-15 BER is required after FEC correction

- It follows the flow in the IEEE presentation of Anslow_3bs_03_0515

Gray coded PAM-4 bit error rate probabilities



For a single bit in error the noise amplitude is ~0.5 * level spacing
For a 2 bit error the noise amplitude is ~1.5 * level spacing, so = 3 * noise amplitude for a single bit error assuming no linearity error
Probability of 2 bit error is < 1e-20 for 1e-3 BER and <1e-45 for 1e-6 BER, so 2 bit errors can be ignored for the rest of this presentation



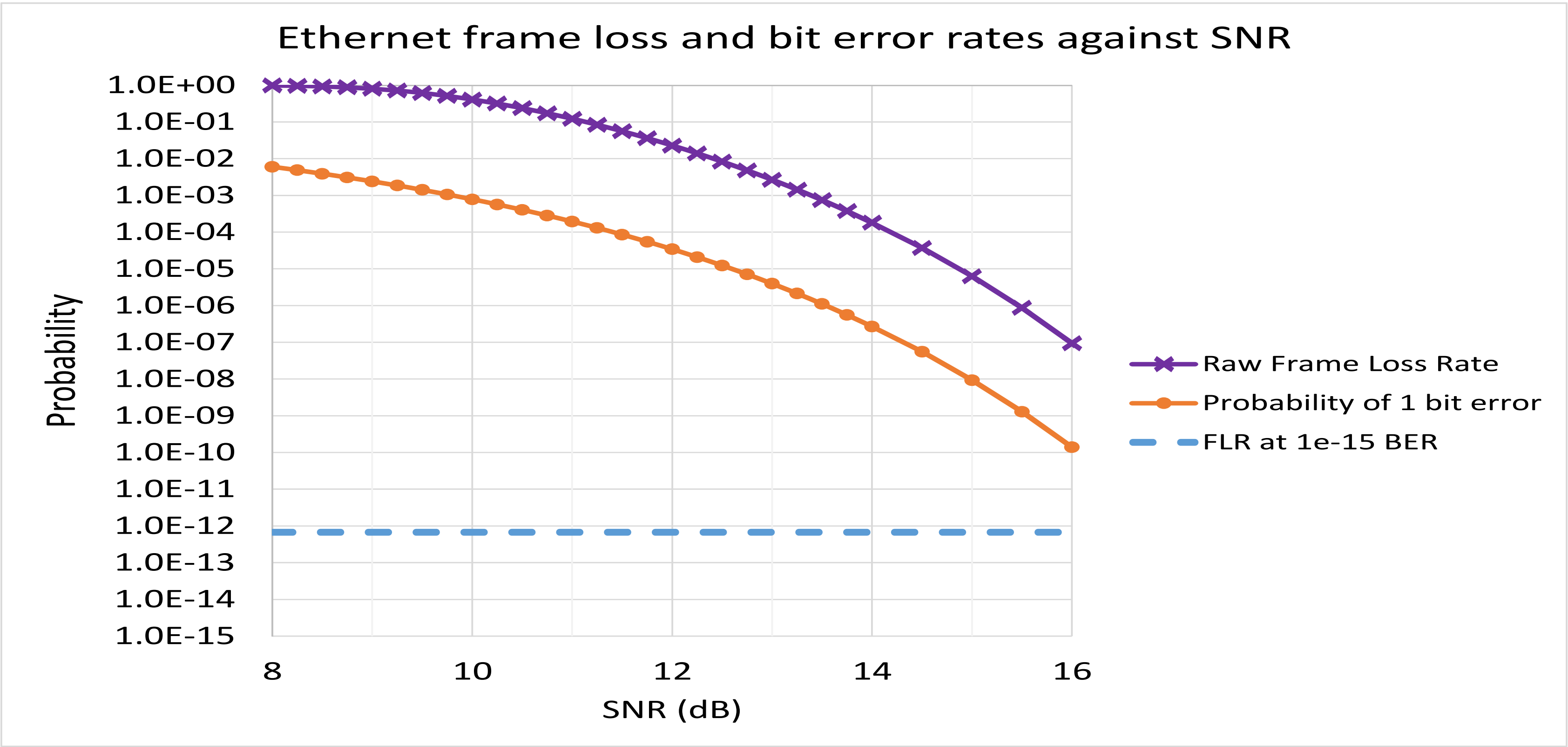
Basic error probability theory

- The value of the Signal to Noise Ratio (SNR) used in the following curves is derived from the equation for a random noise induced error:

$$BER_{in} = \frac{1}{2} erfc\left(\frac{SNR}{\sqrt{2}}\right)$$

- Where SNR is the signal to noise ratio expressed as a ratio and not in dB
- The minimum Ethernet frame length is (64 + 8 + 12) * 8 = 672 bits and is used to correlate results with the Anslow presentation
- If the probability of a bit error is BER_{in} then the probability of a bit not being in error is $(1 - BER_{in})$
- Therefore, if we assume no error propagation, then the probability of a frame of 672 bits having at least one error is $1 - (1 - BER_{in})^{672}$ and is the Frame Loss Ratio (FLR)

Random errors with no error propagation with no FEC



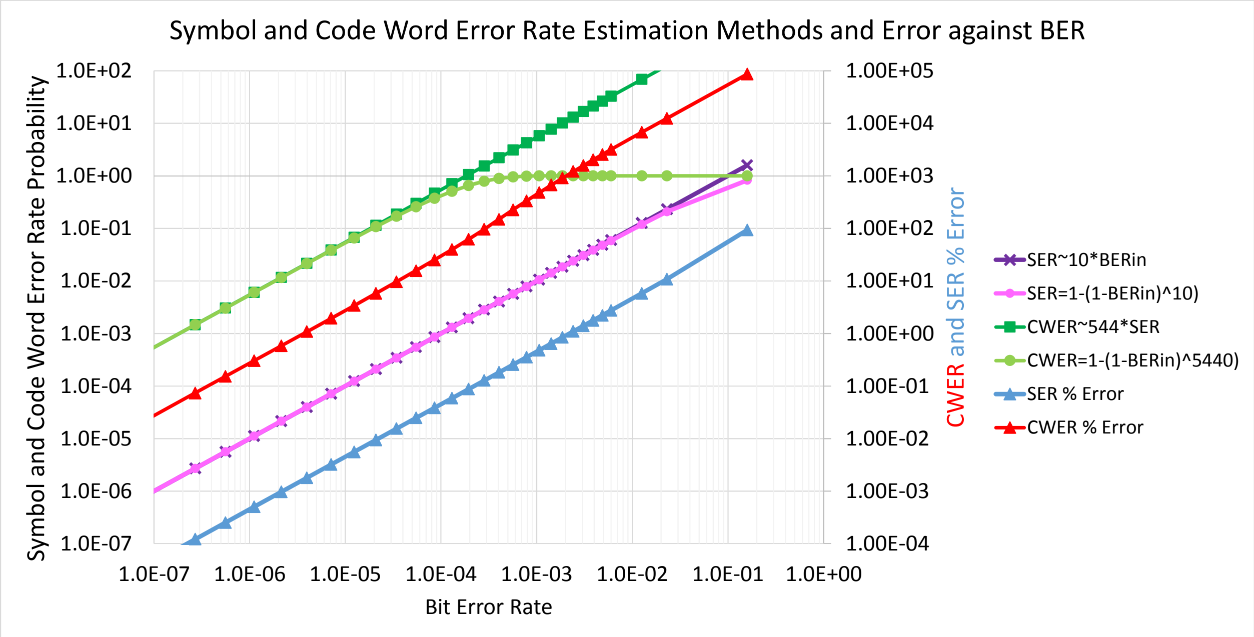
(544,514,10) RS FEC error correction capability

- Block length n = 544
- Message length k = 514
- Symbol length = 10
- Hamming distance = $n - k + 1$ = 31
- Erroneous symbols in a block that can be corrected = 0.5 * ($n - k$) = 15

FEC mathematics with no error propagation

- For a BER of BER_{in} the probability of 1 bit not having an error is$(1 - BER_{in})$
- The probability of a 10 bit symbol not having an error is $(1 - BER_{in})^{10}$
- The probability of a 10 bit symbol having any number of errors is:

$(1 - (1 - BER_{in})^{10})$ = SER, for a very small BER_{in} this reduces to $10 * BER_{in}$



How small can we use the approximation?
As we have 544 symbols per code word the probability of no errors in a code word is
(1-Symbol Error Rate)^{544}
The simple approximation would be 5440*BER and only gives less than 1% error for BER < ~3e-6 and >100% error for 3e-4 and above, so not good for the BER range we need to consider

SER = Symbol error rate
CWER = Code word error rate

FEC mathematics with no error propagation – continued

- **The probability of an error free symbol $P(0) = 1 - SER$**
- **The probability of 544 error free symbols $P(544, no\ errors) = (1 - SER)^{544}$**

- **The probability of one errored symbol $P(1) = 1 - (1 - BER_{in})^{10} = SER$**
- **Number of ways of picking one errored symbol from 544 symbols $= 544$**
- **The probability of one errored symbol in 544 symbols $P(1, 544) = 544 * SER$**
- **The probability of 543 symbols with no errors $P(543, no\ errors) = (1 - SER)^{543}$**
- **The probability of one errored symbol and 543 non-errored symbols**

$$P(1\ error\ only) = 544 * SER * (1 - SER)^{543}$$

- **The probability of two errored symbols in 2 symbols $P(2) = SER^2$**
- **Number of ways of picking 2 symbols from 544 $= \frac{544!}{(544-2)! * 2!}$**
- **Probability of 2 errored symbols in 544 $= \frac{544! * SER^2}{(544-2)! * 2!}$**
- **The probability of *542* symbols with no errors $P(542, no\ errors) = (1 - SER)^{542}$**
- **The probability of two errored symbols and 542 non-errored symbols**

$$P(2\ errors\ only) = \frac{544! * SER^2}{(544-2)! * 2!} * (1 - SER)^{542}$$

FEC mathematics with no error propagation – continued 2

- **The probability of n errored symbols $P(n) = SER^n$**
- **Number of ways of picking n errored symbols from 544 $= \frac{544!}{(544-n)!n!}$**
- **The probability of *544-n* symbols with no errors**

  $$P(544 - n, no\ errors) = (1 - SER)^{544-n}$$
- **Probability of n errored symbols in 544 $= \frac{544!*SER^n}{(544-n)!*n!}$**
- **The probability of n errored symbols and *544-n* non-errored symbols**

  $$P(n\ errors\ only) = \frac{544!*SER^n}{(544-n)!*n!} * (1 - SER)^{544-n}$$

FEC mathematics with no error propagation –continued 3

- **The probability of up to 15 errored symbols**

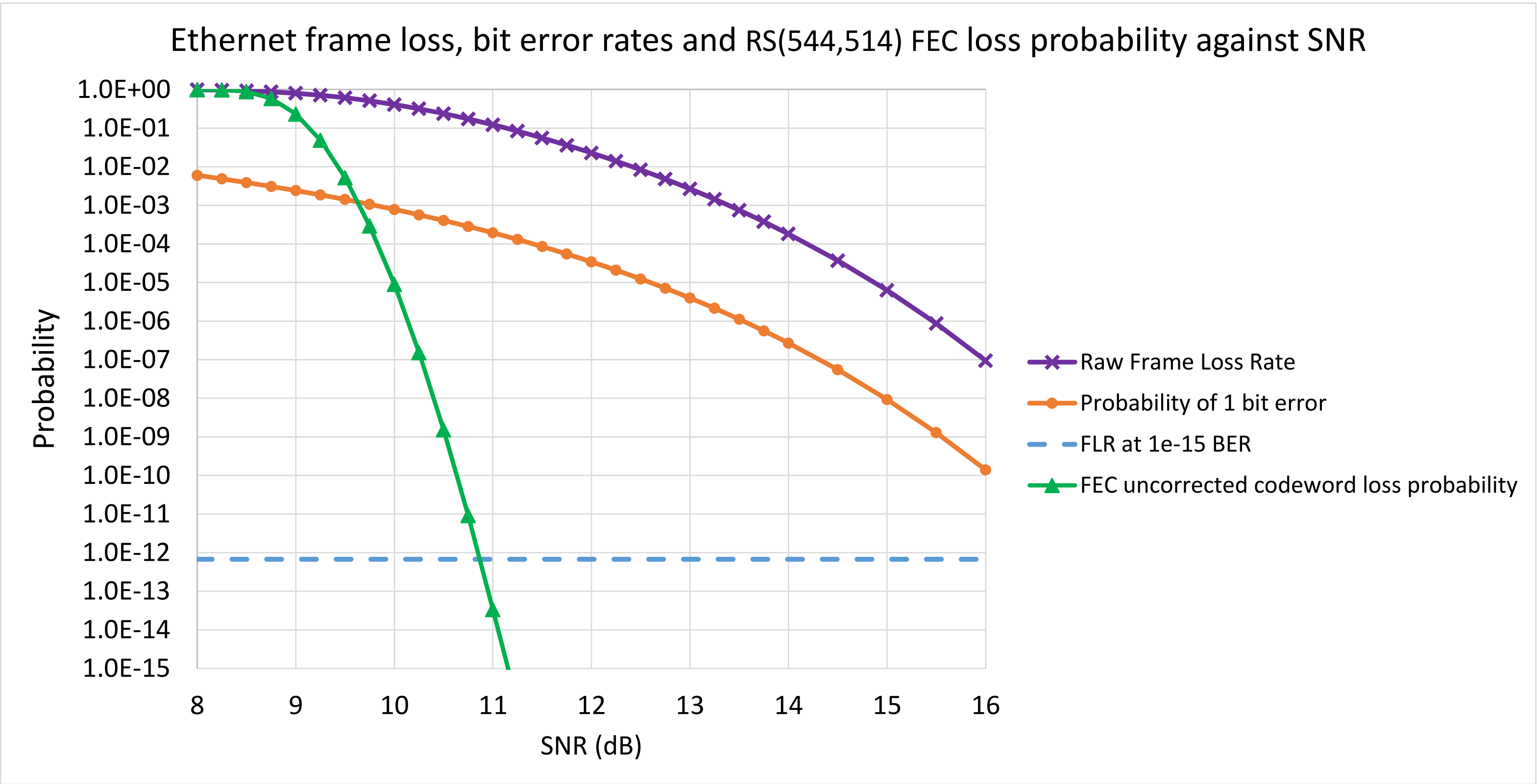
$$P(0to15\,errors) = \sum_{n=0}^{n=15} P(n\,error\,only) = \sum_{n=0}^{n=15} \frac{544! * SER^{n}}{(544-n)! * n!} * (1-SER)^{544-n}$$

- **All of the 0 to 15 errored symbols cases will be corrected by the FEC, so the probability of an uncorrected error will be**

$$P(FECuncorrected) = (1 - P(0to15\,errors))$$

$$P(FECuncorrectable) = 1 - \sum_{n=0}^{n=15} \frac{544! * SER^{n}}{(544-n)! * n!} * (1-SER)^{544-n}$$

Random errors with no error propagation with and without FEC

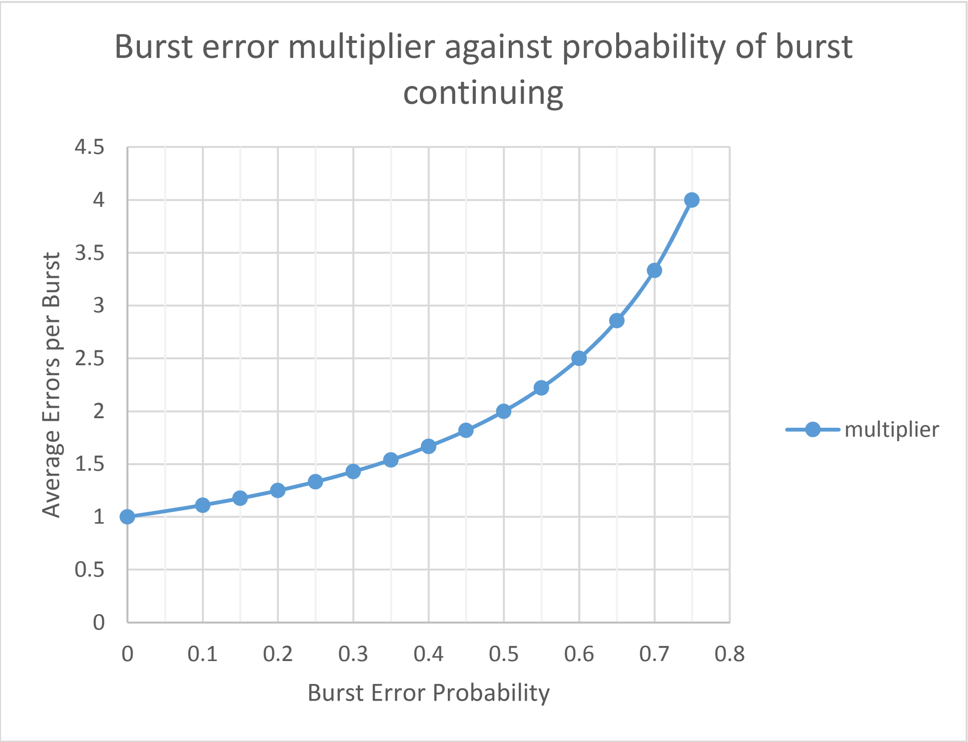


Effect of Burst Error Probability on BER

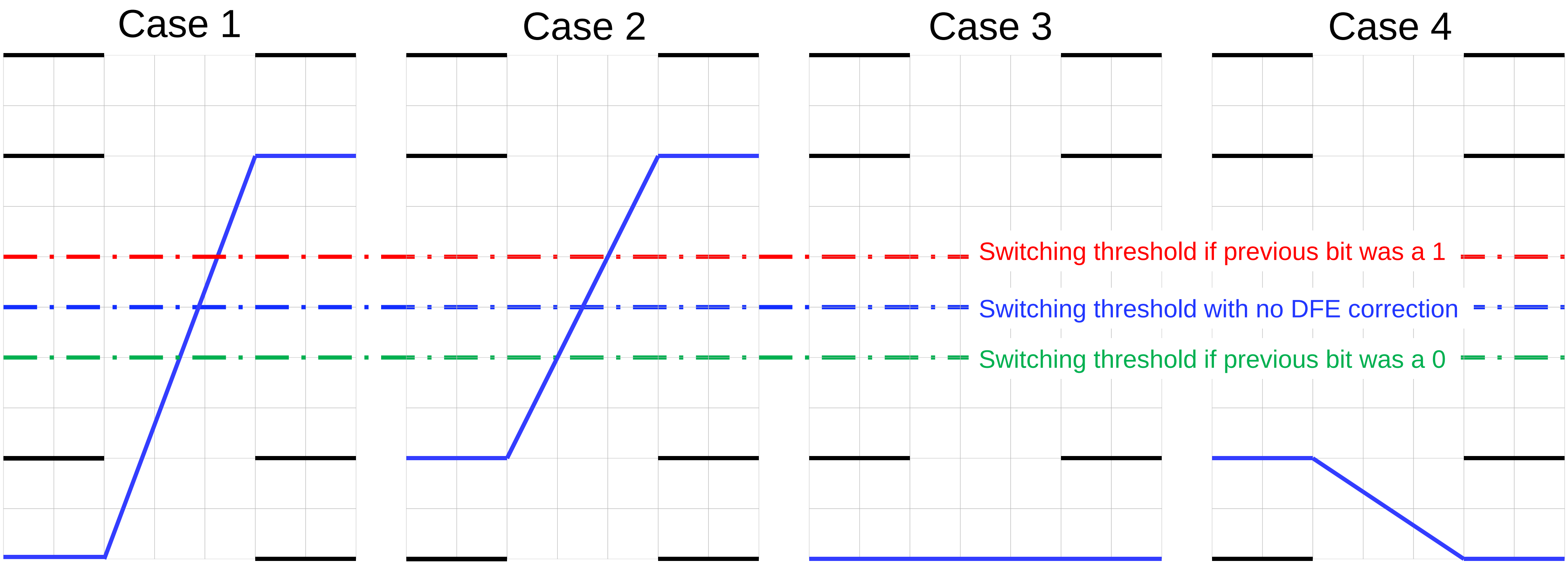
- The average number of errors in a burst is simply related to the probability of an error propagating (A)

$$Average\ errors\ per\ burst = \frac{1}{1-A}$$

- However, with PAM-4 there are only 5 PAM-4 symbols per 10 bit FEC symbol
- For an error in the last bit of one FEC symbol with PAM-4 to propagate into the next symbol is very high, but to propagate to the following FEC symbol after that is also high and needs to be considered



NRZ Error propagation options for 1 DFE tap only



Case 1	Case 2	Case 3	Case 4
0 to 1 (prior bit 0)	0 to 1 (prior bit 1)	0 to 0 (prior bit 0)	0 to 0 (prior bit 1)
If correct margin = 4	If correct margin = 4	If correct margin = 4	If correct margin = 4
If incorrect margin = 2	If incorrect margin = 2	If incorrect margin = 6	If incorrect margin = 6

In 50% of all cases the margin is reduced by 2 * DFE tap 1 value and could cause an error to propagate

Worst Case PAM-4 Error Propagation

Correct level	Perceived RX level		Errored bits	
	One higher	One lower	One higher	One lower
3 (1,0)	3 (1,0)	2 (1,1)	✓, ✓	✓,×
2 (1,1)	3 (1,0)	1 (0,1)	✓,×	×, ✓
1 (0,1)	2 (1,1)	0 (0,0)	×, ✓	✓,×
0 (0,0)	1 (0,1)	0 (0,0)	✓,×	✓, ✓

Two of these cases result in ✓, ✓ so the error will not propagate (25%)
This means a 75% chance of a PAM-4 error propagating
Therefore, average burst length = 4 PAM-4 symbols
Two of these cases result in errors in the first bit (25%), four cases in the second bit (50%)

FEC Symbol Error Propagation for PAM-4 Errors

The probability of an error in the first PAM-4 symbol affecting the second PAM-4 symbol is 0.75, the third PAM-4 symbol is 0.75^2, the fourth is 0.75^3, the fifth 0.75^4 and the first PAM-4 symbol in the next FEC symbol is 0.75^5.

This can be done for the second, third, fourth and fifth PAM-4 symbols being in error and gives the total probability of an error in one FEC symbol causing the next FEC symbol to also be in error of:

$$P_{err}(next\ FEC\ symbol) = \frac{(0.75^5 + 0.75^4 + 0.75^3 + 0.75^2 + 0.75^1)}{5} = 0.4576$$

This gives a 46% chance of the next FEC symbol being in error due to a burst of errors

The alarming extension of this is that the next but one FEC symbol being in error has a probability given by:

$$P_{err}(next\ but\ one\ FEC\ symbol) = \frac{(0.75^{10} + 0.75^9 + 0.75^8 + 0.75^7 + 0.75^6)}{5} = 0.1086$$

$P_{err}(next\ but\ one\ FEC\ symbol) = 0.75^5 * P_{err}(next\ but\ one\ FEC\ symbol)$ Note $0.75^5 = 0.2373$

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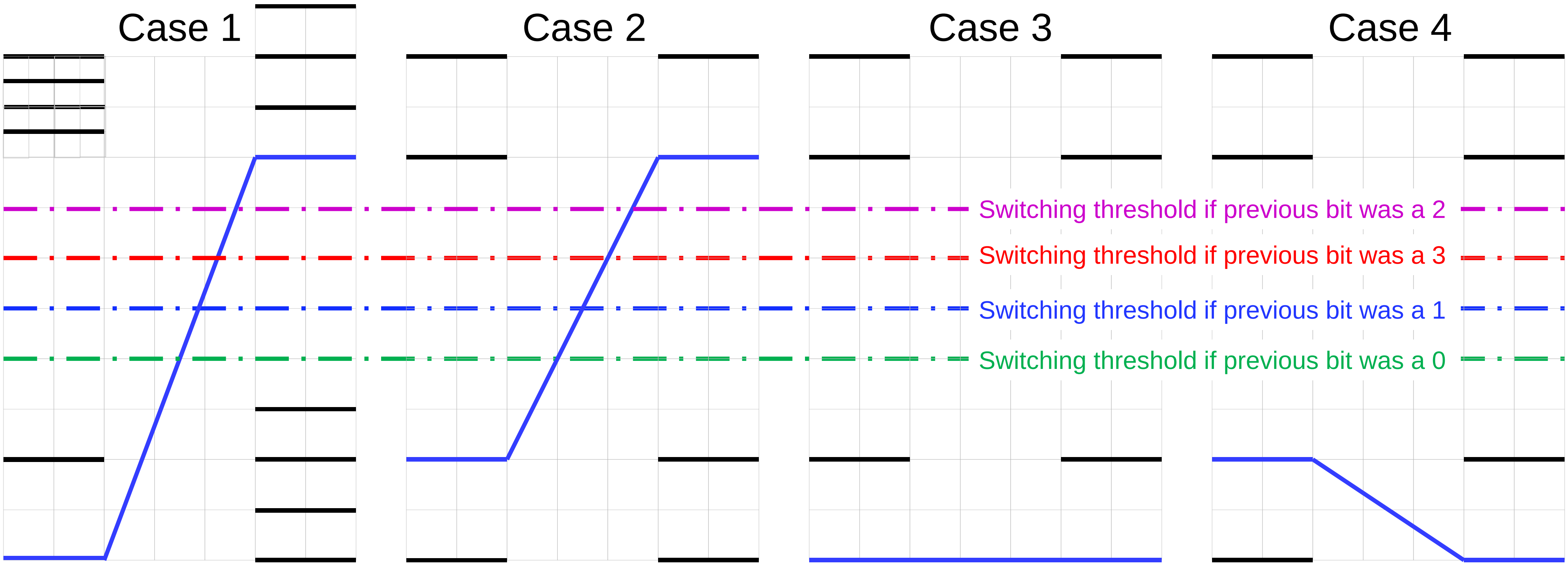
PAM-4 Error propagation options for 1 DFE tap only



0 can only error to 1
If correct margin = n
If incorrect margin = n-1

1 can error to 0 or 3
If correct margin = n
If 0 margin = 2
If 3 margin =

0 to 0 (prior bit 0)
If correct margin = 4
If incorrect margin = 6

0 to 0 (prior bit 1)
If correct margin = 4
If incorrect margin = 6

In 50% of all cases the margin is reduced by 2 * DFE tap value and could cause an error to propagate